

Instanton, Monopole Condensation and Confinement

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The confinement mechanism in the nonperturbative QCD is studied in terms of topological excitation as QCD-monopoles and instantons. In the 't Hooft abelian gauge, QCD is reduced into an abelian gauge theory with monopoles, and the QCD vacuum can be regarded as the dual superconductor with monopole condensation, which leads to the dual Higgs mechanism. The monopole-current theory extracted from QCD is found to have essential features of confinement. We find also close relation between monopoles and instantons using the lattice QCD. In this framework, the lowest 0^{++} glueball ($1.5 \sim 1.7\text{GeV}$) can be identified as the QCD-monopole or the dual Higgs particle.

1. Dual Higgs Theory for NP-QCD

Quantum chromodynamics (QCD) is established as the strong-interaction sector in the Standard Model, and the perturbative QCD provides the powerful and systematic method in analyzing high-energy experimental data. However, QCD is a 'black box' in the infrared region still now owing to the strong-coupling nature, although there appear rich phenomena as *color confinement*, *dynamical chiral-symmetry breaking* and *topological excitation* in the nonperturbative QCD (NP-QCD). In particular, confinement is the most outstanding feature in NP-QCD, and to understand the confinement mechanism is a central issue in hadron physics.

In 1974, Nambu [1] presented an interesting idea that quark confinement and string picture for hadrons can be interpreted as the squeezing of the color-electric flux by the *dual Meissner effect*, which is similar to formation of the Abrikosov vortex in the type-II superconductor. This *dual superconductor picture* for the NP-QCD vacuum is based on the *duality* in the Maxwell equation, and needs condensation of *color-magnetic monopoles*, which is the dual version of electric-charge (Cooper-pair) condensation in the superconductivity.

In 1981, 't Hooft [2] pointed out that color-magnetic monopoles appear in QCD as topological excitation in the *abelian gauge* [2,3], which diagonalizes a gauge-dependent variable $X(s)$.

Here, $SU(N_c)$ gauge degrees of freedom is partially fixed except for the maximal torus subgroup $U(1)^{N_c-1}$ and the Weyl group. In the abelian gauge, QCD is reduced into a $U(1)^{N_c-1}$ -gauge theory, and *monopoles with unit magnetic charge* appear at hedgehog-like configurations according to the nontrivial homotopy group, $\Pi_2\{SU(N_c)/U(1)^{N_c-1}\} = Z_\infty^{N_c-1}$ [4-6].

In 90's, the Monte Carlo simulation based on the lattice QCD becomes a powerful tool for the analysis of the confinement mechanism using the maximally abelian (MA) gauge [7-15], which is a special abelian gauge minimizing the off-diagonal components of the gluon field. Recent lattice studies with MA gauge have indicated monopole condensation in the NP-QCD vacuum [7-9] and the relevant role of abelian degrees of freedom, *abelian dominance* [9-12], for NP-QCD. In the lattice QCD in MA gauge, *monopole dominance* for NP-QCD is also observed as the essential role of QCD-monopoles for the linear quark potential [10], chiral symmetry breaking [11,12] and instantons [6,13,14].

In this paper, we study QCD-monopoles in the NP-QCD vacuum using the $SU(2)$ lattice QCD in MA gauge. Next, we study the role of QCD-monopoles to quark confinement using the monopole-current theory extracted from QCD [16]. Finally, we study the correlation between instantons and QCD-monopoles in terms of remaining nonabelian nature in MA gauge.

2. Lattice QCD in MA Gauge

The maximally abelian (MA) gauge is the best abelian gauge for the dual superconductor picture for NP-QCD. In this section, we consider the mathematical structure of MA gauge. In the SU(2) lattice formalism, MA gauge is defined so as to maximize

$$\begin{aligned} R &\equiv \sum_{s,\mu} \text{tr}\{U_\mu(s)\tau_3 U_\mu^{-1}(s)\tau_3\} \\ &= 2 \sum_{s,\mu} \{U_\mu^0(s)^2 + U_\mu^3(s)^2 - U_\mu^1(s)^2 - U_\mu^2(s)^2\} \end{aligned} \quad (1)$$

by the gauge transformation. Here, $U_\mu(s) \equiv \exp\{ia e A_\mu(s)\} \equiv U_\mu^0(s) + i\tau^a U_\mu^a(s)$ denotes the link-variable on the lattice with spacing a . In MA gauge in the lattice formalism,

$$\begin{aligned} X[U_\mu(s)] &\equiv \sum_{\mu=1}^4 U_{\pm\mu}(s) \tau_3 U_{\pm\mu}^{-1}(s) \\ &= \sum_{\mu=1}^4 \{U_\mu(s)\tau_3 U_\mu^{-1}(s) + U_\mu^{-1}(s - \hat{\mu})\tau_3 U_\mu(s - \hat{\mu})\} \end{aligned} \quad (2)$$

is diagonalized. Here, we use the convenient notation as $U_{-\mu}(s) \equiv U_\mu^{-1}(s - \hat{\mu})$.

In MA gauge, there remain U(1)₃-gauge symmetry and global Weyl symmetry [13], because R is invariant under the gauge transformation $U_\mu(s) \rightarrow v(s)U_\mu(s)v^{-1}(s + \hat{\mu})$ with $v(s) = e^{i\tau^3\phi^3(s)} \in \text{U}(1)_3$ and the Weyl transformation $U_\mu(s) \rightarrow WU_\mu(s)W^{-1}$ with $W \in \text{Weyl}_2 \simeq Z_2$ being s -independent. W is expressed as

$$\begin{aligned} W &\equiv \exp\{i\pi(\frac{\tau_1}{2} \cos \phi + \frac{\tau_2}{2} \sin \phi)\} \\ &= i(\tau_1 \cos \phi + \tau_2 \sin \phi) = i \begin{pmatrix} 0 & e^{-i\phi} \\ e^{i\phi} & 0 \end{pmatrix} \end{aligned} \quad (3)$$

and interchanges SU(2)-quark color, $|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. In the SU(N_c) case, this Weyl symmetry Weyl_{N_c} corresponds to the permutation group $P_{N_c} = Z_{N_c(N_c-1)/2}$, whose element interchanges SU(N_c)-quark color [13,17].

Nonabelian gauge symmetry $G \equiv \text{SU}(N_c)_{\text{local}}$ is reduced into $H \equiv \text{U}(1)_{\text{local}}^{N_c-1} \times \text{Weyl}_{N_c}^{\text{global}}$ in MA

gauge. Then, the independent set of the gauge function $\Omega_{\text{MA}}(s)$ which realizes MA gauge fixing corresponds to the coset space G/H : $\Omega_{\text{MA}}(s) \in G/H$. The representative element for the link-variable in MA gauge is expressed as $U_\mu^{\text{MA}}(s) \equiv \Omega_{\text{MA}}(s)U_\mu(s)\Omega_{\text{MA}}^{-1}(s + \hat{\mu})$, and also forms G/H . Thus, MA gauge fixing obeys the nonlinear representation on coset space G/H .

The MA gauge function $\Omega_{\text{MA}}(s) \in G/H$ is transformed nonlinearly by $g(s) \in G \equiv \text{SU}(N_c)$ as $\Omega_{\text{MA}}(s) \rightarrow \Omega_{\text{MA}}^g(s) = h[g](s)\Omega_{\text{MA}}(s)g^{-1}(s)$, where $h[g](s) \in H$ appears so as to satisfy $\Omega_{\text{MA}}^g(s) \in G/H$. Actually, the successive gauge transformation, Ω_{MA}^g after g , is equivalent to $h[g]\Omega_{\text{MA}}$, and maximizes R .

According to the nonlinear transformation in $\Omega_{\text{MA}} \in G/H$, any operator \hat{O}_{MA} defined in MA gauge transforms nonlinearly as $\hat{O}_{\text{MA}} \rightarrow \hat{O}_{\text{MA}}^{h[g]}$ by the SU(N_c)-gauge transformation $g \in G$.

(Proof) For simplicity, original \hat{O} is assumed to obey the adjoint transformation by the SU(N_c)-gauge transformation. Then, one finds $\hat{O}_{\text{MA}} = \Omega_{\text{MA}}\hat{O}\Omega_{\text{MA}}^{-1}$. By $g \in G$, \hat{O}_{MA} is transformed as $\hat{O}_{\text{MA}} \rightarrow \hat{O}_{\text{MA}}^g = \Omega_{\text{MA}}^g\hat{O}\Omega_{\text{MA}}^{g^{-1}} = h[g]\Omega_{\text{MA}}\hat{O}\Omega_{\text{MA}}^{-1} \cdot g\hat{O}g^{-1} \cdot g\Omega_{\text{MA}}h[g]^{-1} = h[g]\hat{O}_{\text{MA}}h[g]^{-1} = \hat{O}_{\text{MA}}^{h[g]}$ with $h[g] \in H$. This proof can be generalized to any operator \hat{O} .

If \hat{O}_{MA} is H -invariant, one gets $\hat{O}_{\text{MA}}^{h[g]} = \hat{O}_{\text{MA}}$ for any $h[g] \in H$, so that \hat{O}_{MA} is invariant under arbitrary gauge transformation by $g \in G$. Thus, one finds a useful criterion on the SU(N_c)-gauge invariance of the operator in MA gauge [13].

“If an operator \hat{O}_{MA} defined in MA gauge is H -invariant, \hat{O}_{MA} is proved to be also invariant under the whole gauge transformation of G .”

3. Abelian/Monopole Projection

The SU(2) link-variable $U_\mu(s)$ can be factorized as $U_\mu(s) = M_\mu(s)u_\mu(s)$, where $u_\mu(s) \equiv \exp\{i\tau^3\theta_\mu^3(s)\} \in \text{U}(1)_3$ is *abelian link-variable* and $M_\mu(s) \equiv e^{i\tau^1\theta_\mu^1(s)+\tau^2\theta_\mu^2(s)} \in \text{SU}(2)/\text{U}(1)_3$,

$$\begin{aligned} M_\mu(s) &\equiv \exp\{i\tau^1\theta_\mu^1(s) + \tau^2\theta_\mu^2(s)\} \\ &\equiv \begin{pmatrix} \cos \theta_\mu(s) & -e^{-i\chi_\mu(s)} \sin \theta_\mu(s) \\ e^{i\chi_\mu(s)} \sin \theta_\mu(s) & \cos \theta_\mu(s) \end{pmatrix} \end{aligned} \quad (4)$$

with $-\pi < \theta_\mu^3(s), \chi_\mu(s) \leq \pi$ and $0 \leq \theta_\mu(s) \leq \frac{\pi}{2}$. Here, $\cos \theta_\mu(s)$ in the abelian gauge is a gauge-invariant quantity which measures the ‘U(1)-ratio’ of the link-variable $U_\mu(s)$. For instance, $\langle \cos \theta_\mu(s) \rangle = 1$ means perfectly abelian system.

In MA gauge, the off-diagonal component of $U_\mu^{\text{MA}}(s)$ is strongly suppressed as $M_\mu(s) \simeq 1$ or $U_\mu^{\text{MA}}(s) \simeq u_\mu(s)$. Actually, the SU(2) lattice QCD in MA gauge shows high ‘U(1)-ratio’ as $\langle \cos \theta_\mu(s) \rangle_{\text{MA}} \geq 0.9$ even in the strong-coupling region. Therefore, QCD in MA gauge becomes similar to the abelian gauge theory.¹

For any operator $\hat{O}[U_\mu(s)]$, *abelian projection* is realized as $\langle \hat{O}[U_\mu(s)] \rangle \rightarrow \langle \hat{O}[u_\mu(s)] \rangle_{\text{MA}}$. In case of $\langle \hat{O}[U_\mu(s)] \rangle \simeq \langle \hat{O}[u_\mu(s)] \rangle_{\text{MA}}$, the abelian degrees of freedom is relevant for $\hat{O}[U_\mu(s)]$ in MA gauge, which is called as *abelian dominance* for \hat{O} . For instance, the SU(2) lattice QCD shows abelian dominance [10] for the string tension as $\langle \sigma[u_\mu(s)] \rangle_{\text{MA}} \simeq 0.92 \cdot \langle \sigma[U_\mu(s)] \rangle$.

In U(1)₃ link-variable $u_\mu(s) = \exp\{i\tau^3 \theta_\mu^3(s)\}$, $\theta_\mu^3(s) \in (-\pi, \pi]$ is the abelian gauge field on the lattice, and *abelian field strength* is defined as

$$\theta_{\mu\nu}^{\text{FS}}(s) \equiv \text{mod}_{2\pi}(\partial \wedge \theta^3)_{\mu\nu}(s) \in (-\pi, \pi], \quad (5)$$

which is U(1)₃-gauge invariant. Generally, $\theta_\mu^3(s)$ satisfies $(\partial \wedge \theta^3)_{\mu\nu}(s) = \theta_{\mu\nu}^{\text{FS}}(s) + 2\pi n_{\mu\nu}(s)$. Here, $n_{\mu\nu}(s) \in \mathbf{Z}$ corresponds to the Dirac string and varies by singular U(1)₃-gauge transformation. There appear magnetic-monopole currents

$$k_\nu(s) \equiv \frac{1}{2\pi} \partial_\mu \tilde{\theta}_{\mu\nu}^{\text{FS}}(s) = -\partial_\mu \tilde{n}_{\mu\nu}(s) \in \mathbf{Z} \quad (6)$$

and the electric current $j_\nu(s) \equiv \frac{1}{2\pi} \partial_\mu \theta_{\mu\nu}^{\text{FS}}(s)$.

We show in Fig.1 the monopole current $k_\mu(s)$ in the lattice QCD in MA gauge. In the deconfinement phase, monopole currents only appear as short-range fluctuation. In the confinement phase, monopole currents cover the whole lattice and form a global structure, which is an evidence of *monopole condensation* [7,8].

Nonperturbative phenomena like confinement are brought by large fluctuation of gauge fields in the strong-coupling region. In MA gauge, such large fluctuation is concentrated into the

¹ In the UV region, such a criterion would be meaningless because $U_\mu(s) \rightarrow 1$ as $\beta \rightarrow \infty$ in a suitable gauge.

U(1)₃ sector, $u_\mu(s)$. In particular, monopoles appear at the ends of the Dirac strings, and accompany large fluctuation of $u_\mu(s)$ or $\theta_\mu^3(s)$. Hence, monopole density $\rho_M \equiv \frac{1}{V} \sum_{s,\mu} |k_\mu(s)|$ is expected to measure the magnitude of gauge-field fluctuation. Here, $|k_\mu(s)|$ and ρ_M in MA gauge are SU(N_c)-gauge invariant, since $|k_\mu(s)|$ is U(1)₃-gauge invariant and Weyl₂-invariant.

The abelian gauge field $\theta_\mu^3(s)$ can be decomposed into the *monopole part* $\theta_\mu^{Mo}(s)$ and the *photon part* $\theta_\mu^{Ph}(s)$,

$$\begin{aligned} \theta_\nu^{Mo}(s) &\equiv 2\pi \sum_{s'} \langle s | \partial^{-2} | s' \rangle \partial_\mu n_{\mu\nu}(s'), \\ \theta_\nu^{Ph}(s) &\equiv \sum_{s'} \langle s | \partial^{-2} | s' \rangle \partial_\mu \theta_{\mu\nu}^{\text{FS}}(s'). \end{aligned} \quad (7)$$

In the Landau gauge $\partial_\mu \theta_\mu^3(s) = 0$, one finds $\theta_\mu^3(s) = \theta_\mu^{Mo}(s) + \theta_\mu^{Ph}(s)$. U(1)₃ link-variables are defined as $u_\mu^{Mo,Ph}(s) \equiv \exp\{i\tau_3 \theta_\mu^{Mo,Ph}(s)\}$.

From $\theta_\mu^{Mo}(s)$ and $\theta_\mu^{Ph}(s)$, one can derive the field strength and the currents in the monopole and photon sectors using Eqs.(4) and (5) [11,13]. *The monopole sector holds the monopole current only* : $k_\mu^{Mo}(s) \simeq k_\mu(s)$ and $j_\mu^{Mo}(s) \simeq 0$, while *the photon sector holds the electric current only* : $j_\mu^{Ph}(s) \simeq j_\mu(s)$ and $k_\mu^{Ph}(s) \simeq 0$.

Monopole projection is realized as $\langle \hat{O}[U_\mu(s)] \rangle \rightarrow \langle \hat{O}[u_\mu^{Mo}(s)] \rangle_{\text{MA}}$, and *monopole dominance* as $\langle \hat{O}[U_\mu(s)] \rangle \simeq \langle \hat{O}[u_\mu^{Mo}(s)] \rangle_{\text{MA}}$ is observed for NP-QCD. For instance, monopole dominance for the string tension [10] is observed in the lattice QCD as $\langle \sigma[u_\mu^{Mo}(s)] \rangle_{\text{MA}} \simeq 0.88 \cdot \langle \sigma[U_\mu(s)] \rangle$.

4. Monopole Dynamics for Confinement

In MA gauge, QCD-monopoles seem essential degrees of freedom for NP-QCD. In this section, we investigate monopole dynamics and confinement properties using the monopole-current action [3] extracted from the lattice QCD [16],

$$Z = \sum_{k_\mu(s) \in \mathbf{Z}} \exp\{-\alpha \sum_s k_\mu^2(s)\} \delta(\partial_\mu k^\mu(s)), \quad (8)$$

which is defined on lattices with large spacing a . In the dual Higgs phase, nonlocal interactions between the monopole current would vanish effectively due to the *screening effect* [3].

Here, the monopole current with length L is regarded as L -step *self-avoiding random walk* with

$2d-1=7$ possible direction in each step. Hence, the partition function (7) is approximated as $Z = \sum_L \rho(L) e^{-\alpha L} \simeq \sum_L e^{-(\alpha - \ln 7)L}$, where the configuration number of monopole loop with length L is estimated as $\rho(L) \simeq 7^L$. In this system, the *Kosterlitz-Thouless-type transition* occurs at $\alpha_c = \ln 7$ similarly in vortex dynamics in the 2-dimensional superconductor.

We perform direct simulations of partition function (7) on lattices. Fig.2 shows monopole density $\rho_M \equiv \frac{1}{V} \sum_{s,\mu} |k_\mu(s)|$ and the clustering parameter $\eta \equiv \sum_i L_i^2 / (\sum_i L_i)^2$ as functions of self-energy α . As α increases, ρ_M decreases monotonously, and *declustering of monopole current* is observed around $\alpha_c = 1.8 \simeq \ln 7$.

Now, we study confinement in the monopole-current system using the dual field formalism [4,5,17]. We introduce the dual gauge field B_μ satisfying $\tilde{F}_{\mu\nu} = (\partial \wedge B)_{\mu\nu}$. In the dual Landau gauge $\partial_\mu B^\mu = 0$, one finds $\partial^2 B_\mu = k_\mu$. Hence, starting from the monopole current configuration $k_\mu(x)$, the dual gauge field is derived as

$$B_\mu(x) = \partial^{-2} k_\mu(x) = -\frac{1}{4\pi^2} \int d^4 y \frac{k_\mu(y)}{(x-y)^2}, \quad (9)$$

which leads $F_{\mu\nu}$ and the Wilson loop. The Wilson loop $\langle W \rangle$ shown in Fig.3 obeys the area law. We show in Fig.4 the string tension σa^2 as the function of α . Similar behavior is found between ρ_M and σa^2 , which suggests the relevant role of monopoles for confinement.

Thus, the monopole theory (7) seems to have essence of NP-QCD in the infrared region. In real QCD, however, the QCD-monopole would have *its intrinsic size* $R \sim 0.3\text{fm}$ [3], for it is a *collective mode* composed by gluons. Hence, *monopole size effects* should appear in the UV region, $a \leq R$, and the monopole action (7) is modified to be *nonlocal*. In fact, the monopole size R may provide a *critical scale* for NP-QCD in term of the dual Higgs theory.

5. Instantons and QCD-monopoles

The instanton is another relevant topological object in QCD according to $\Pi_3(\text{SU}(N_c)) = \mathbb{Z}_\infty$. Recent studies reveal close relation between instantons and QCD-monopoles [6,8,13-15,18,19].

In Fig.5, we show the lattice QCD result for the linear correlation between the total monopole-loop length L and $I_Q \equiv \frac{1}{16\pi^2} \int d^4 x |\text{tr}(G_{\mu\nu} \tilde{G}_{\mu\nu})|$, which corresponds to the total number of instantons and anti-instantons. The lattice QCD shows also *monopole dominance for instantons* [13,14]: $\langle I_Q[U_\mu] \rangle \simeq \langle I_Q[M_\mu u_\mu^{M\sigma}] \rangle_{\text{MA}}$ and $\langle I_Q[M_\mu u_\mu^{Ph}] \rangle_{\text{MA}} \simeq 0$ after several cooling. Hence, instantons can be regarded as ‘seeds’ of QCD-monopoles [6,8,10,13,14,18,19].

For these correlation, off-diagonal elements would be essential. On the SU(2) lattice in MA gauge, we find relatively large off-diagonal elements remaining around monopoles. Hence, instantons, which need full SU(2) components, appear near monopole world-lines in MA gauge.

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FIGURE CAPTION

Fig.1. Monopole current in MA gauge extracted from SU(2) lattice QCD with $16^3 \times 4$. (a) confinement phase ($\beta = 2.2$), (b) deconfinement phase ($\beta = 2.4$).

Fig.2. (a) Monopole density ρ_M v.s. self-energy α . (b) The clustering parameter η v.s. α . For $\alpha < \alpha_c$, almost monopole world-lines combine into one large cluster ($\eta \simeq 1$). For $\alpha > \alpha_c$, only small monopole loops appear ($\eta \simeq 0$).

Fig.3. The Wilson loop $\langle W(I, J) \rangle$ v.s. $I \times J$ in the monopole theory with $\alpha=1.7, 1.8, 1.9$.

Fig.4. The string tension σa^2 in the monopole-current theory. The dotted line denotes the Creutz ratio in the lattice QCD with $\beta=1.25\alpha$.

Fig.5. Correlation between I_Q and the total monopole-loop length L in the SU(2) lattice QCD. We plot the data at 3 cooling sweep on the $16^3 \times 4$ lattice with various β .